integral®

Comprehensive, high-quality support for Cambridge International AS/A level Mathematics and Further Mathematics

HODDER EDUCATION

Can be used alongside **Hodder Education's** range of textbooks and eTextbook

era 2020 FINALIST

Developed by MEI Mathematics Education Innovation

Introducing **Integral**

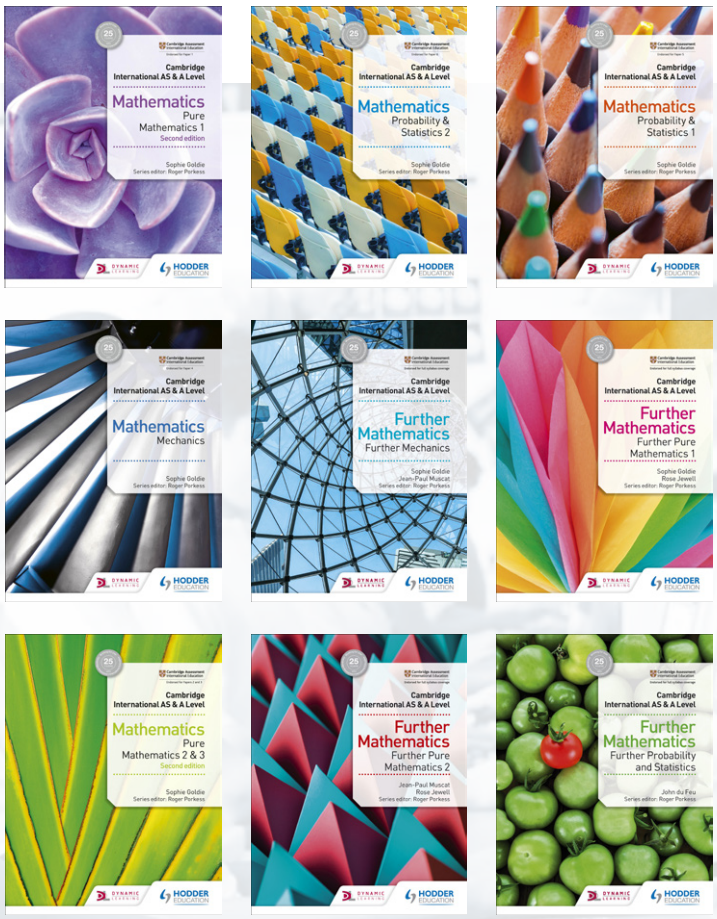
The Integral® online teaching and learning platform is the ideal companion to Hodder Education's range of textbooks and eTextbooks for Cambridge International AS/A level Mathematics (Syllabus 9709) and Further Mathematics (Syllabus 9231).

Integral provides high-quality teaching and learning activities to enhance the use of these textbooks. It covers the whole of each syllabus and includes printable materials, innovative interactive activities, and exercises with worked solutions and automatically marked on-screen tests.

Integral links seamlessly with the eTextbooks and is easy to use alongside the printed textbooks

Integral is produced by MEI, an independent charity that has been supporting mathematics education for more than 55 years. As a result of a unique collaboration between MEI and Hodder Education, Integral links seamlessly with the eTextbooks and is easy to use alongside the printed textbooks.

Integral is available by subscription. Find out more at:
integralmaths.org/cambridge_international



Why **Integral?**

Integral helps teachers to make best use of their time, allowing them to focus on planning, teaching and reviewing students' progress.

- Enhances textbooks and eTextbooks, providing everything teachers and students need
- Easy to use on computers and tablets
- Designed by experienced specialists to develop deep mathematical understanding
- Tailored to the Cambridge International syllabuses
- Used by over 30,000 students and 6,000 teachers in the UK alone
- Covers the whole curriculum with thousands of teaching and learning resources

Using Integral with Hodder Education eTextbooks

Hodder Education's eTextbooks are fully integrated with Integral, so that all of the relevant resources are readily accessible when needed, whether at the front of the class or planning lessons. Provided the user has access to both Hodder Education eTextbooks and Integral, they can move with ease between corresponding topics.

Here is an example from the eTextbook for **Pure Mathematics 1**

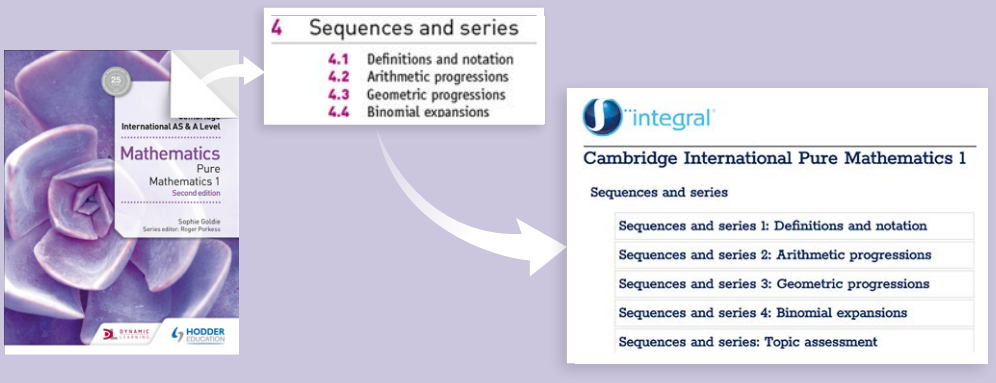
You can easily get to the eTextbooks by clicking the eTextbook link within Integral.

You can also access the relevant Integral resources from within the pages of the eTextbook.

Using Integral with Hodder Education printed textbooks

The content of Integral is structured to align with the structure of the printed textbooks, so it is easy to find the Integral resources that go with each chapter of the textbook

Here is an example from the printed textbook for **Pure Mathematics 1**



Over 100 sections
with extensive content

Integral has over 100 sections covering the content of AS/A level Mathematics and Further Mathematics. Each section is laid out as in the example below

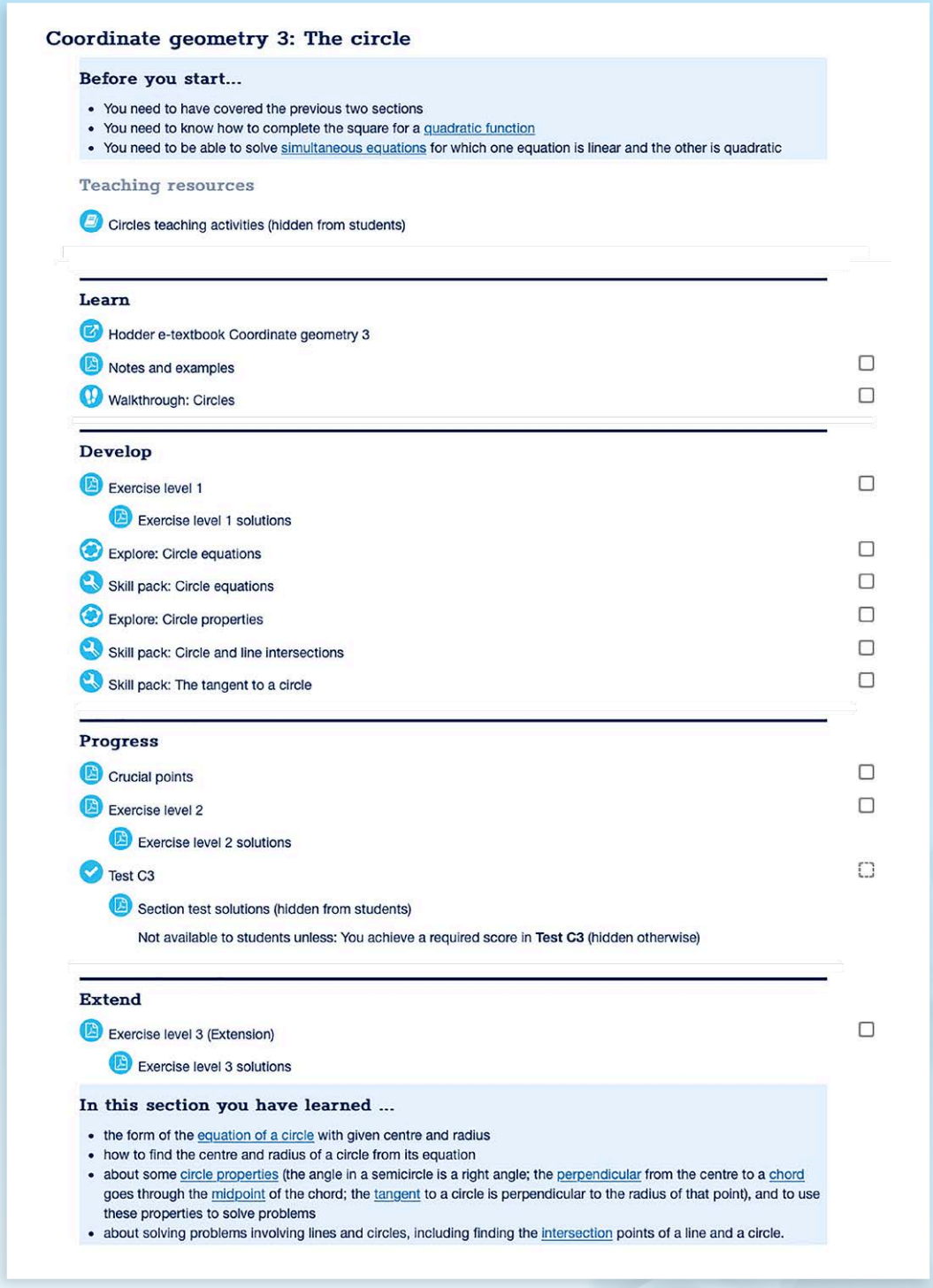
The **Teaching resources** area is packed with resources to use in the classroom

The **Learn** area introduces material new for this section

The **Develop** area is a place to practice the new methods and formulae

The **Progress** area provides challenge at a level similar to that in examinations and gives immediate feedback via the on-screen section test

The **Extend** area provides extra challenge beyond that required for exams



Coordinate geometry 3: The circle

Before you start...

- You need to have covered the previous two sections
- You need to know how to complete the square for a quadratic function
- You need to be able to solve simultaneous equations for which one equation is linear and the other is quadratic

Teaching resources

Circles teaching activities (hidden from students)

Learn

Hodder e-textbook Coordinate geometry 3

Notes and examples ☐

Walkthrough: Circles ☐

Develop

Exercise level 1 ☐

Exercise level 1 solutions

Explore: Circle equations ☐

Skill pack: Circle equations ☐

Explore: Circle properties ☐

Skill pack: Circle and line intersections ☐

Skill pack: The tangent to a circle ☐

Progress

Crucial points ☐

Exercise level 2 ☐

Exercise level 2 solutions

Test C3 ☐

Section test solutions (hidden from students)

Not available to students unless: You achieve a required score in **Test C3** (hidden otherwise)

Extend

Exercise level 3 (Extension) ☐

Exercise level 3 solutions

In this section you have learned ...

- the form of the equation of a circle with given centre and radius
- how to find the centre and radius of a circle from its equation
- about some circle properties (the angle in a semicircle is a right angle; the perpendicular from the centre to a chord goes through the midpoint of the chord; the tangent to a circle is perpendicular to the radius of that point), and to use these properties to solve problems
- about solving problems involving lines and circles, including finding the intersection points of a line and a circle.

Section tests
help students prepare for exams

Section tests contain short and long questions and cover both method and problem solving

Question 3
Not complete
Marked out of 3.00
Flag question
Edit question

The equation of a circle with centre (2, 1) and radius 6 is

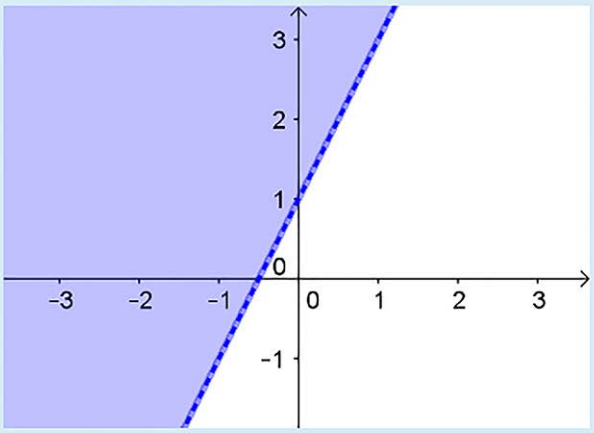
Select one:
- $(x+2)^2+(y+1)^2=36$
- $(x-2)^2+(y-1)^2=6$
- $(x+2)^2+(y+1)^2=6$
- $(x-2)^2+(y-1)^2=36$

Check

Over **100 section tests** cover the whole syllabus

Question 10
Not complete
Marked out of 3.00
Flag question
Edit question

Which of the inequalities describes the shaded region in the diagram?



Select one:
- $y < 2x + 1$
- $y > 2x + 1$
- $y \leq 2x + 1$
- $y \geq 2x + 1$

Check

Students' **responses are tracked** and conveniently accessible to teachers

Question 10
Not complete
Marked out of 3.00
Flag question
Edit question

The line $y = 2x$ does not meet the circle $(x-2)^2 + (y-1)^2 = d$.

The range of possible values for d are given by d [▾] [ ]

(Give your answer as an exact whole number or a tion in its lowest terms in the form p/q).

Check

<
>
≤
≥

Powerful analytics for section tests

At a glance information about students' responses in Section Tests, with a red/ amber/green system

Compare your students to the average score of all Integral users

Click in a cell to see the **detailed student response** to that question

First name / Surname	Grade/100.00	Q. 1 /10.00	Q. 2 /10.00	Q. 3 /10.00	Q. 4 /10.00	Q. 5 /10.00	Q. 6 /10.00	Q. 7 /10.00	Q. 8 /10.00	Q. 9 /10.00	Q. 10 /10.00
Demo Student2 Review attempt	**80.00**	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✗ 0.00	✓ 10.00	✓ 10.00	✗ 0.00
Demo Student1 Review attempt	**100.00**	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00	✓ 10.00
Demo Student3 Review attempt	**53.33**	✓ 10.00	✗ 0.00	✓ 10.00	✓ 10.00	✗ 0.00	✓ 3.33	✓ 10.00	✗ 0.00	✗ 0.00	✓ 10.00
Demo Student4 Review attempt	**0.00**	✗ 0.00	✗ 0.00	✗ 0.00	✗ 0.00	✗ 0.00	✗ 0.00	✗ 0.00	✗ 0.00	✗ 0.00	✗ 0.00
Demo Student5 Review attempt	**25.00**	✗ 0.00	✗ 0.00	✗ 0.00	✓ 10.00	✗ 0.00	✓ 5.00	✓ 10.00	✗ 0.00	✗ 0.00	✗ 0.00
Overall average	**51.67 (5)**	6.00 (5)	4.00 (5)	6.00 (5)	8.00 (5)	4.00 (5)	5.67 (5)	6.00 (5)	4.00 (5)	4.00 (5)	4.00 (5)

Student5

Quiz Test P2

Question Notation6

Completed on Tuesday, 8 March 2020, 4:27 PM

Question 6

Partially correct

Mark 1.50 out of 3.00

Which of the following is a counter example to the statement $(x+3)^2 > (x-1)^2$ for all values of x?

Select one or more:

- $x = 1$
- $x = 0$
- ✓ $x = -1$
- $x = -2$

Your answer is partially correct.

You have correctly selected 1.

The correct answers are: $x = -1$, $x = -2$

Partially correct

Marks for this submission: 1.50/3.00.

High-quality, thorough and accessible notes and examples

Quick links to access the content covered

Cambridge International

Pure Mathematics 1 Coordinate geometry

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Section 3: The circle

Notes and Examples

These notes and examples contain subsections on

- The equation of a circle
- Finding the equation of a circle
- Circle geometry
- The intersection of a line and a curve
- The intersection of two curves

Helpfully written, extensive notes with lots of fully worked examples with additional comments

The intersection of a line and a curve

Just as the point of intersection of two straight lines can be found by solving the equations of the two lines simultaneously, the point(s) of intersection of a line and a curve can be found by solving their equations simultaneously.

In many cases, the equations of both the line and the curve are given as an expression for y in terms of x. When this is the case, a sensible first step is to equate the expressions for y, as this leads to an equation in x only.

Example 6

Find the coordinates of the points where the line $y = x + 2$ meets the curve $y = x^2 - 3x + 5$.

Solution

$x^2 - 3x + 5 = x + 2$

Equate the expressions for y to give an equation in x only

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3$ or $x = 1$

When $x = 3$ then $y = 3 + 2 = 5$

When $x = 1$ then $y = 1 + 2 = 3$.

Substitute the x values into the equation of the line

The points where the line meets the curve are (3, 5) and (1, 3).

You should check that each of these points satisfies the equation of the curve. (You have already used the equation of the line to find the y-values).

Interactive learning 'walkthroughs' with dynamic geometry

Interactive pages to introduce key ideas

Help from Pythagoras

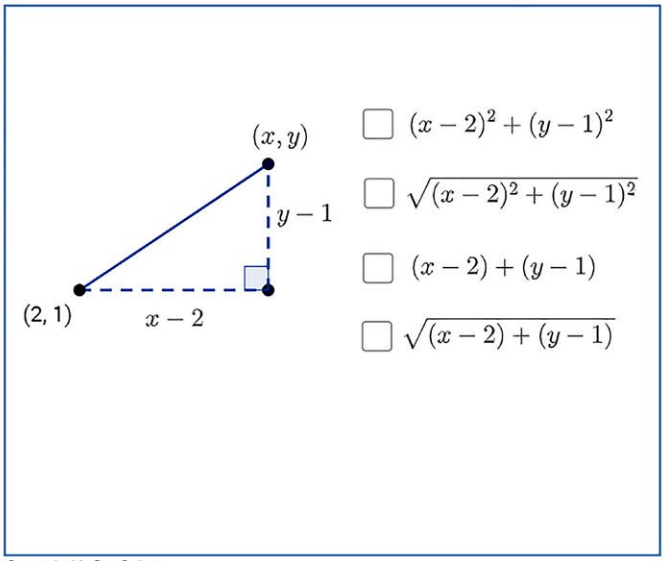
3.

To decide whether a point (x, y) is on the circle with radius 5 centred at $(2, 1)$ you need to work out how far (x, y) is from $(2, 1)$.

Instructions

☑ Tick the expression on the right that gives you the distance between (x, y) and $(2, 1)$.

The diagram will help you with this.



Created with GeoGebra

Submit Reset

Dynamic geometry to help understanding

Adjusting the centre

5.

Just as the equation of the circle centred at $(2, 1)$ with radius 5 is:

$$(x-2)^2 + (y-1)^2 = 25,$$

the equation of the circle with centre at (a, b) with radius 5 is:

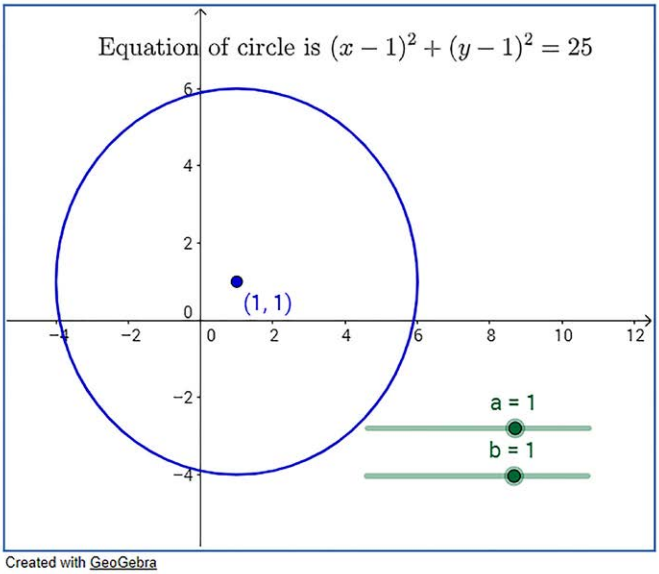
$$(x-a)^2 + (y-b)^2 = 25$$

Instructions

Use the ⊷ sliders to change the values of a and b.

Watch carefully how the centre of the circle changes and how the equation of the circle changes.

Leave the circle so that it has centre at $(0, 0)$.



Created with GeoGebra

Submit Reset

Written exercises, at three levels to stage progress

Method and formulae are tested in the level 1 exercise

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Pure Mathematics 1 Coordinate geometry

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Section 3: The circle

Exercise level 1

1. Find, in the form $x^2 + y^2 + px + qy = c$, the equation for each of the following circles.
   (i) centre (0, 0), radius 6
   (ii) centre (3, 1), radius 5
   (iii) centre (-2, 5), radius 1
   (iv) centre (0, -4), radius 3

Level 2 exercises feature questions **at examination level**

Exercise level 2

1. (i) Show that the line $y = 4 - x$ is a tangent to the circle $x^2 + y^2 = 8$.
   (ii) Show that the line $4y = 3x - 25$ is a tangent to the circle $x^2 + y^2 = 25$.

2. A circle passes through the points Q(0, 3) and R(0, 9) and touches the x-axis. Work out two possible equations for the circle.

3. The line $2y + x = 10$ meets the circle $x^2 + y^2 = 65$ at P and Q. Calculate the length of PQ.

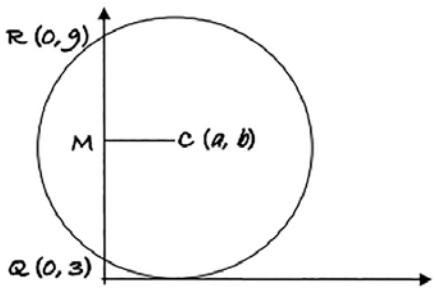
Level 3 exercises provide **extra challenge**

Exercise level 3 (Extension)

1. Find k so that point P $\left(3, \sqrt{27}\right)$ lies on the circle $x^2 + y^2 = k^2$. If P, Q, and R lie on the circle, and triangle PQR is equilateral, write down the coordinates of the two vertices Q and R.

2. (i) P is point (2, 1) and Q is (10, 5). Find the midpoint M of PQ, and hence write down the equation of the circle with PQ as diameter.
   (ii) Line L_1 has equation $y = 3x - 15$. Find the points U, V where line L_1 intersects the circle. What is the angle PUQ?
   (iii) Line L_2 has equation $y + 2x = 5$. Point R lies on line L_2. Find angle RPQ.

Full written solutions to all exercise questions



The midpoint M of QR is (0, 6).
Since a diameter which passes through M is perpendicular to QR, then the line CM

Whole topic
reviews and revision

Topic assessments are ideal for exam preparation and revision

Cambridge International

Pure Mathematics 1 Coordinate geometry

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Topic assessment

1. A line l_1 has equation $5y + 4x = 3$.
   (i) Find the gradient of the line. [1]
   (ii) Find the equation of the line l_2 which is parallel to l_1 and passes through the point (1, -2). [3]

2. Describe fully the curve whose equation is $x^2 + y^2 = 4$. [2]

3. The coordinates of two points are A (-1, -3) and B (5, 7). Calculate the equation of the perpendicular bisector of AB. [4]

Teachers can access **full worked solutions** to topic assessments

Solutions to topic assessment

1. (i) $5y + 4x = 3$.

$5y = -4x + 3$

$y = -\frac{4}{5}x + \frac{3}{5}$

Gradient of line $= -\frac{4}{5}$

[1]

(ii) l_2 is parallel to l_1, so it has gradient $-\frac{4}{5}$.

Equation of line is $y - (-2) = -\frac{4}{5}(x - 1)$

$5(y + 2) = -4(x - 1)$

$5y + 10 = -4x + 4$

$5y + 4x + 6 = 0$

[3]

2. The curve is a circle, centre O and radius 2.

[2]

3. Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{-3 - 7}{-1 - 5} = \frac{-10}{-6} = \frac{5}{3}$

Gradient of line perpendicular to AB $= -\frac{3}{5}$.

The line passes through the midpoint of AB $= \left(\frac{-1 + 5}{2}, \frac{-3 + 7}{2}\right) = (2, 2)$

Equation of line is $y - 2 = -\frac{3}{5}(x - 2)$

$5(y - 2) = -3(x - 2)$

$5y - 10 = -3x + 6$

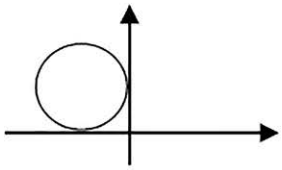
Teaching activities and ideas
for the classroom

Activity 2: exploring circle equations

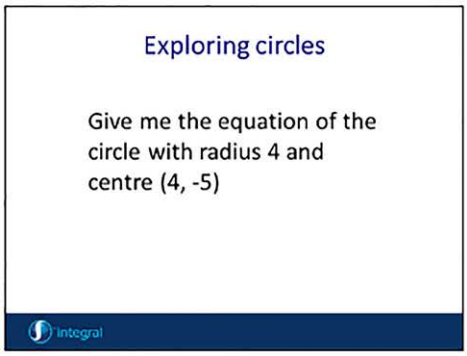
Use the Integral resource Explore: Circle equations, or a graph-drawing package, to explore equations of the form $(x-a)^2+(y-b)^2=r^2$. Show a series of circles with different radii and centres with their equations in completed square form. Give an A3 sheet to each group and ask them to record the equation, the centre and the radius for each circle that comes on to the board. Ask students to come up with patterns that they can then generalise and explain using Pythagoras' theorem.

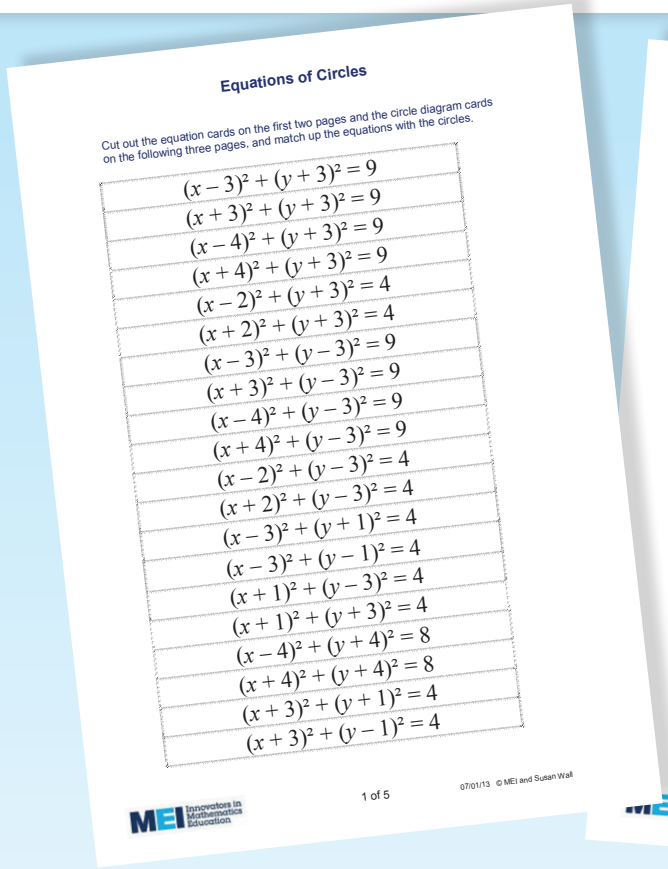
Using mini-whiteboards, ask questions such as

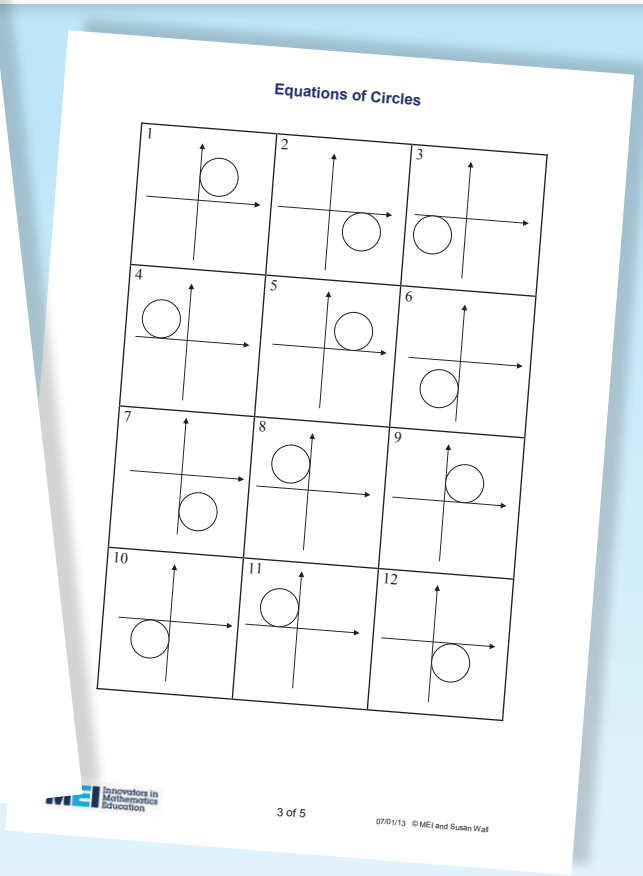
- Give me the equation of the circle with radius 4 and centre (4, -5)
- Give me the equation of a circle with radius 6
- Give me the equation of a circle with centre (-1, 3)
- Give me the equations of two circles with the same centre
- Give me the equations of two circles with the same radius
- Give me a possible equation for this circle



PowerPoint slides for questions above







Integral subscription packages are excellent value

Number of student accounts	Cost for subscription (until 30th September 2020)**
10	£280
20	£300
35	£330
50	£360
75	£445
100	£505
125	£555
150	£595
175	£630
200	£655
200+	Please contact us on +44 1225 774 144 or integralresources@mei.org.uk to discuss

**Subscriptions for the Cambridge International specification normally run for one year from when they are taken out.

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hoddereducation.co.uk/cambridgeasalevelmathematics

Access to Integral can be arranged at
integralmaths.org/cambridge_international

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Developed by MEI
Mathematics Education Innovation

Edition 1 – 2020

About MEI and Integral

Integral has been developed by MEI, an independent charity that has been supporting mathematics education in the UK for more than 55 years. We do this through our range of professional development opportunities, innovative resources and qualifications. MEI is a lead partner in the delivery of national education programmes for the UK government and we work with several leading UK universities.